§1.9: Power functions

<u>Warm up</u>: Find $g(f(x))$ where

$f(x) = \frac{1}{x}$ and $g(x) = \frac{x+1}{x+2}$.

$$g(f(x)) = g\left(\frac{1}{x}\right)$$

$$= \frac{\frac{1}{x}+1}{\frac{1}{x}+2} = \frac{\frac{1}{x}+\frac{x}{x}}{\frac{1}{x}+\frac{2x}{x}} = \ldots$$

<u>Def</u>: A function f is called a "power function" if there is $k \in (-\infty, \infty)$ and $p \in (-\infty, \infty)$ such that

$$f(x) = k \cdot x^p.$$

<u>Eg</u>: $f(x) = 2^x$, $g(x) = \left(\frac{1}{2}\right)^x$ not power ; $a(x) = x^2$, $b(x) = 2x^\pi$

<u>Def</u>: A "polynomial function" is a function that can be written in the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

where $a_n, \cdots, a_0 \in (-\infty, \infty)$, $a_n \neq 0$, and n is a non neg. integer.

- n is the "degree" of f
- a_n is the "leading coeff." of f
- a_0 is the "constant coeff" of f

Ex: x^2+1, $2x^3+x+3$

$$x^{1000} + 2x^{999} + 3x^{998} + \cdots + 1000 \cdot x + 1001$$

Ex: Expand $(\underbrace{x^2+1}_{\text{quad}})(\underbrace{x^3+x+2}_{\text{cubic}})$

$$(x^2+1)(x^3+x+2) = x^2x^3 + x^2x + x^2\cdot 2 + x^3 + x + 2$$

[$x^a x^b = x^{a+b}$

$(x^a)^b = x^{ab}$]

$$= x^5 + x^3 + 2x^2 + x^3 + x + 2$$

$$= x^5 + 2x^3 + 2x^2 + x + 2$$

* Rules for exponents, log's, and composition *

§2: Derivatives

§2.1: Instantaneous rate of change

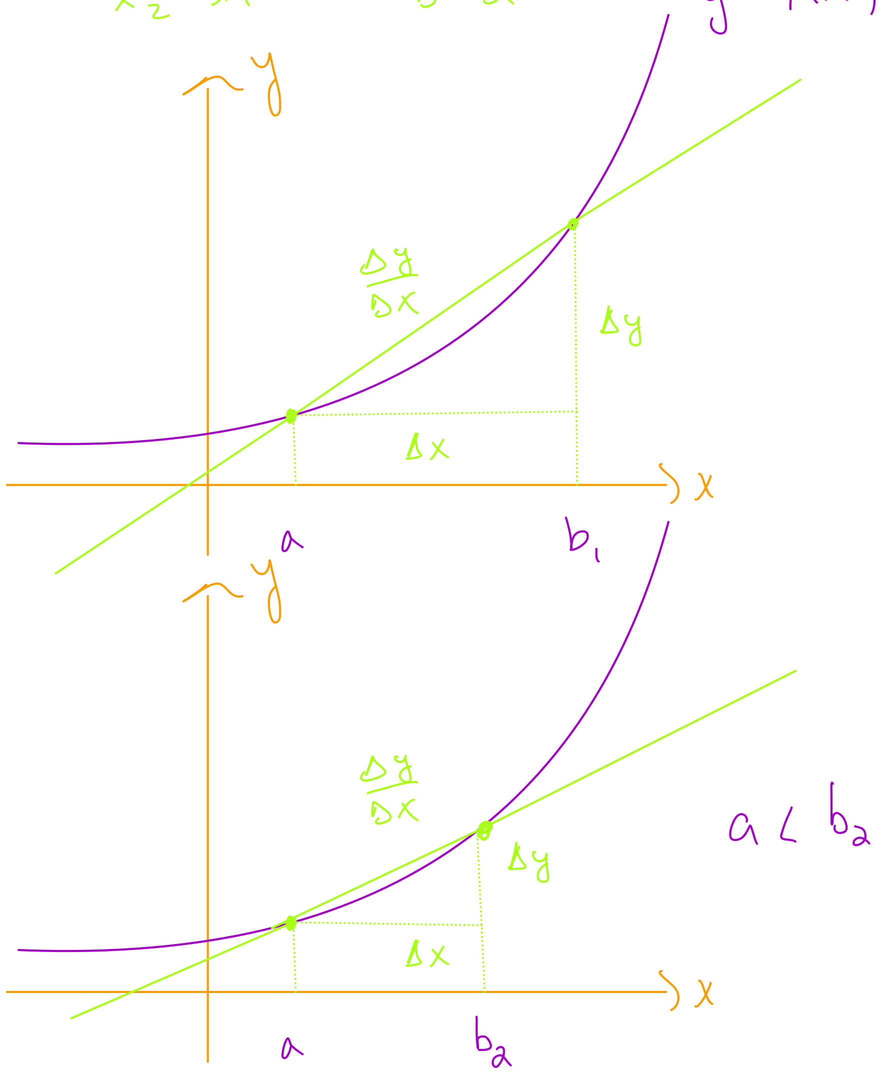
Obj: We study "avg. rate of change" about a pt. as opposed to along an interval.

Rmk: f a function

$[a,b]$ an interval

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(b) - f(a)}{b - a}$$

Rmk:



$a < b_2 < b_1$

Idea: Taking smaller and smaller intervals where b gets closer to a is called "taking the limit".

Def: The "instantaneous rate of change" of a function f at a pt p is defined as the limit of avg. rates of change of f over shorter and shorter intervals about p.

- Denote this value by $f'(p)$, and call it the "derivative" of f at p.

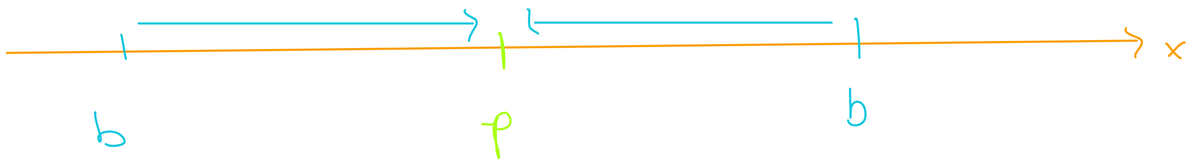
Notation: $f'(p)$ might be also written as $\frac{d}{dx} f \Big|_p$.

Idea:

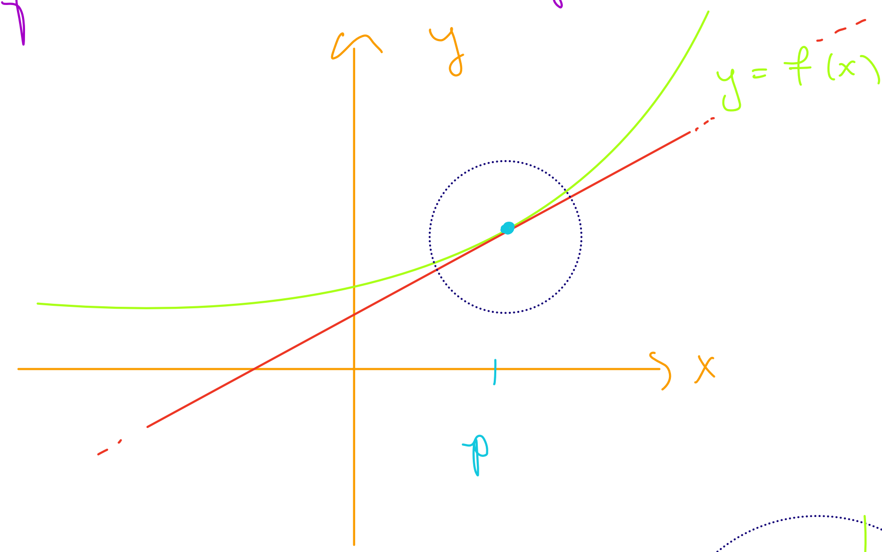
$\frac{\Delta y}{\Delta x}$

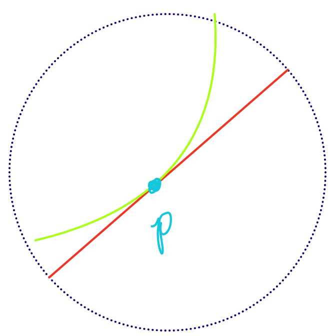
$$f'(p) = \lim. \text{ of } \frac{f(b) - f(p)}{b-p} \text{ as } b \to p$$

where $b \to p$ means b "approaches" p



Rmk: The value $f'(p)$ is the slope of the line tangent to curve at p.





Problems:

§1.8: 5, 6

§1.9: 8, 10, 11

8: $y = \dfrac{5}{2\sqrt{x}} = \dfrac{5}{2}\,\dfrac{1}{\sqrt{x}} \quad \left(= k \cdot x^{p}\right)$

$$= \frac{5}{2} \frac{1}{x^{\frac{1}{2}}}$$

$$= \frac{5}{2} x^{-\frac{1}{2}}$$

$$\left[\frac{1}{x^a} = x^{-a} \right]$$