§2.2: Derivatives

Warm up: Find the average rate of change of the function $f(x) = x^2+1$ from $x = t+a$ to $x = t+b$.

* **Hint**: Think of the wording *

Sol:
$$\frac{\Delta y}{\Delta x} = \frac{f(t+b) - f(t+a)}{(t+b) - (t+a)} = \frac{(t+b)^2+1 - \left((t+a)^2+1\right)}{b-a}$$

$$= \frac{t^2 + 2tb + b^2 + 1 - (t^2 + 2ta + a^2 + 1)}{b-a}$$

$$= \left(\cancel{t^2} + 2tb + b^2 \cancel{+1} - \cancel{t^2} - 2ta - a^2 \cancel{-1}\right)\frac{1}{b-a}$$

$$= \frac{1}{b-a}\left(2tb + b^2 - 2ta - a^2\right)$$

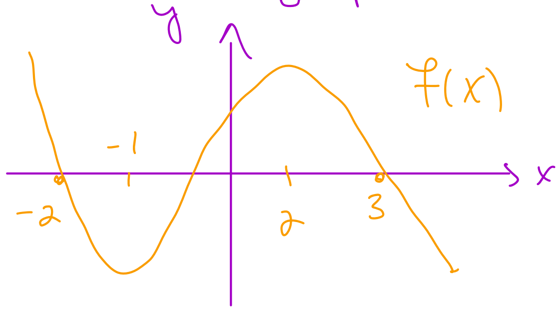
* Compare $\frac{f(b) - f(a)}{b-a}$ *

Def: If f is a function and $a \in (-\infty, \infty)$, then the "derivative" of f at a is

$$f'(a) = \text{inst. rate of change of } f \text{ at } a$$

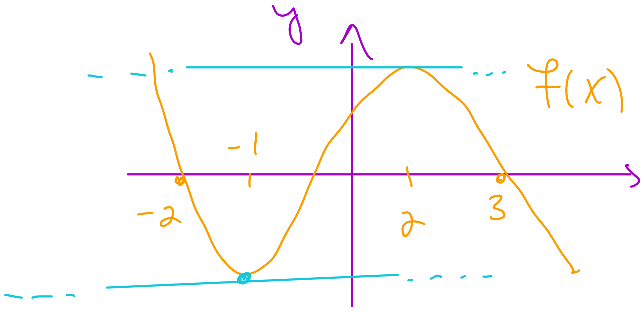
Rmk: $f'(a) =$ slope of tangent line of f at a

Ex: Consider the graph



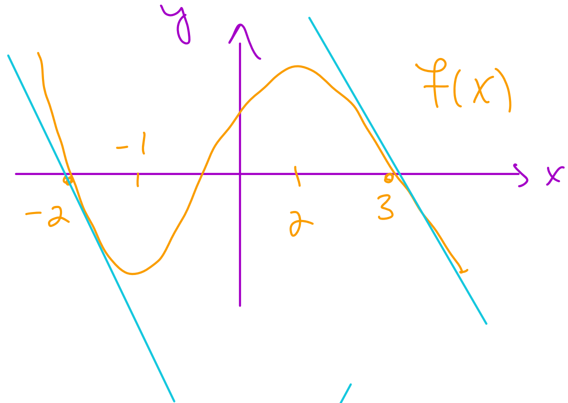
x	$f(x)$	$f'(x)$
0	2	2
1	3	1
2	5	0
⋮		

Can we graph $f'(x)$?

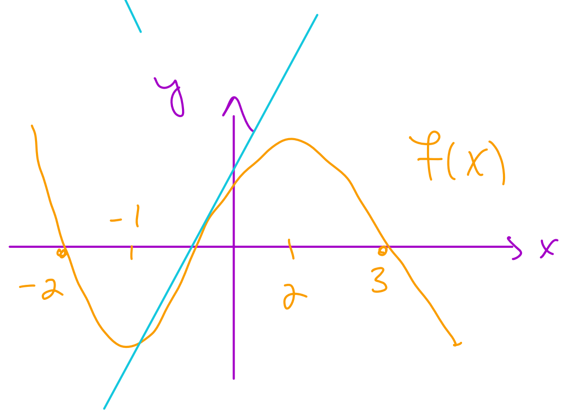


* tangent line at $x = -1, 2$ is zero, is

$f'(-1) = f'(2) = 0$

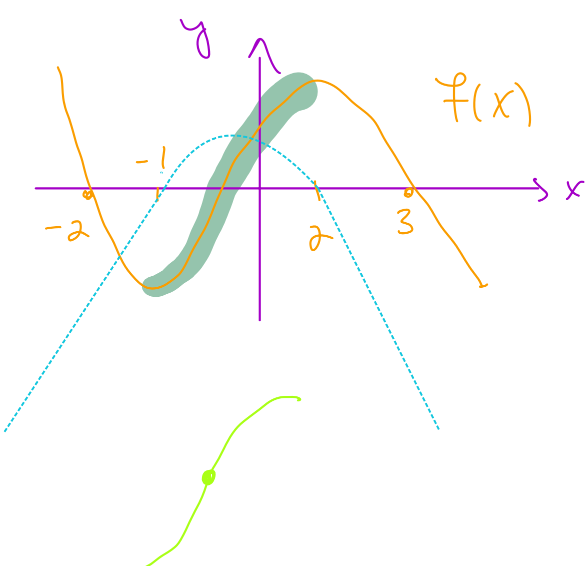


* f is dec. on $x < -1$ and $x > 2$



* f is inc. on $-1 < x < 2$

The graph of $f'(x)$ is

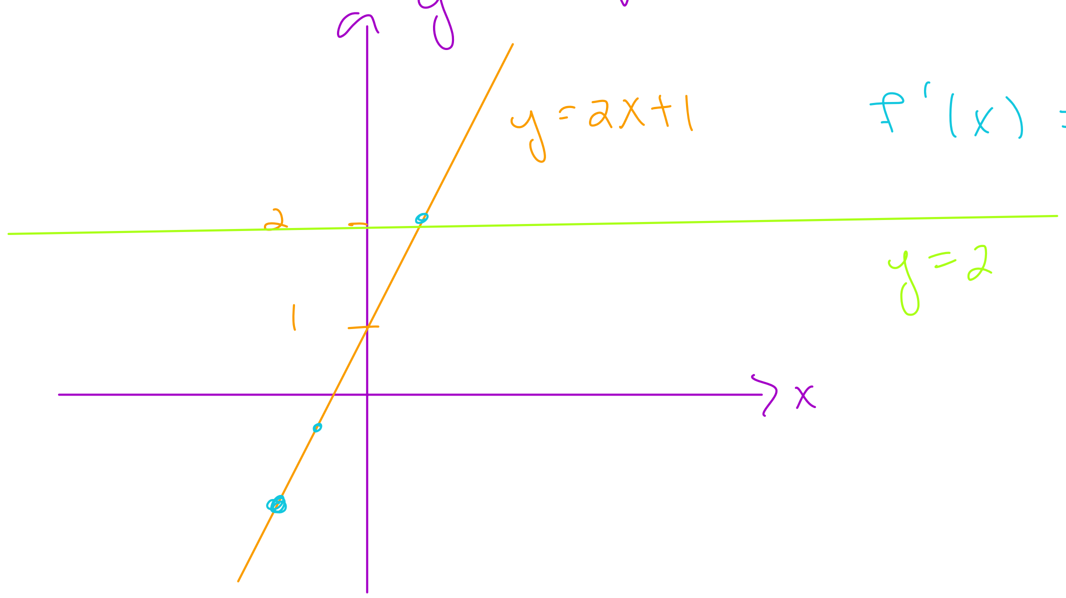


$f'(x)$

Rmk: Let f be a function and I an interval.

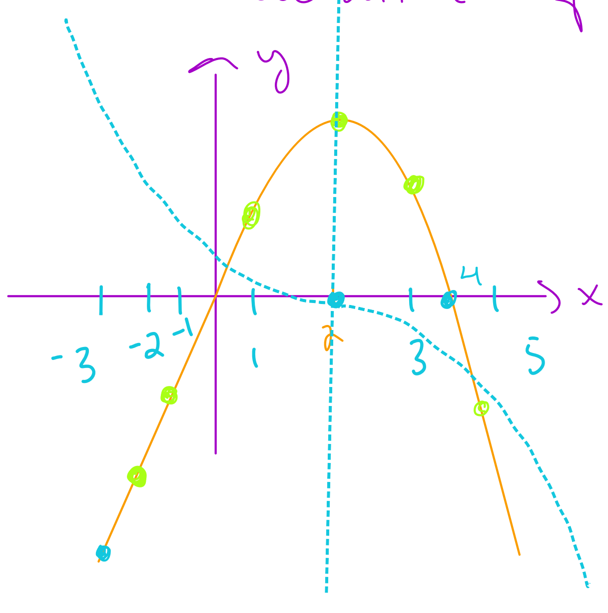
- $f' > 0$ on $I \Rightarrow f$ inc. on I
- $f' < 0$ on $I \Rightarrow f$ dec. on I
- $f' = 0$ on $I \Rightarrow f$ const. on I

Ex: Graph the derivative of $f(x) = 2x+1$



$f'(x) = 2$

Ex: Graph the derivative of the given function



$y = -(x-2)^2 + 4$

§2.3: Interpretations of der.

* Please read on your own *

Rmk: (Tangent line approx.) If $y = f(x)$ and Δx is near zero, then $\Delta y \approx f'(x)\,\Delta x$. For x near a, we have $\Delta y = f(x) - f(a)$ and $\Delta x = x - a$, so

$$f(x) \approx f(a) + f'(a)\,\Delta x$$

Rmk: We will return later to make the above more precise w/ computations.

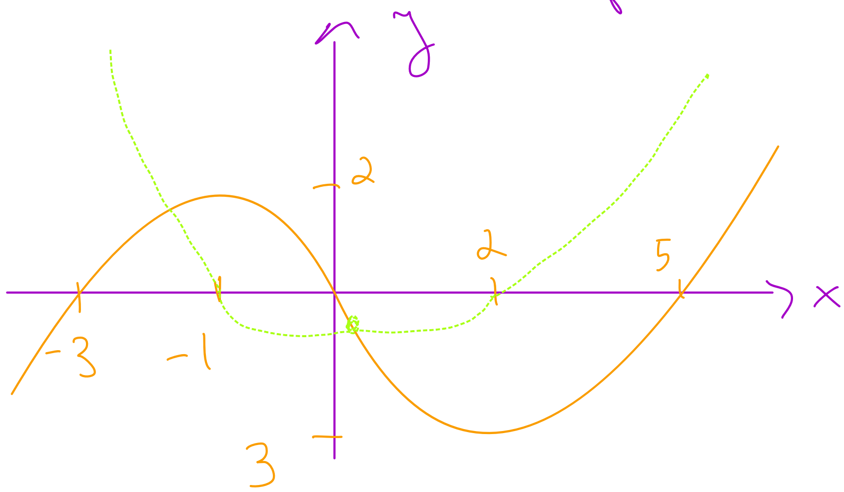
§2.2: (cont.)

Ex: Estimate $f'(1)$, $f'(2)$, and $f'(3)$

where $f(x) = x^2$.

* Hint: Avg. rate of change about a pt. *

Q1: Graph the derivative of the given function.



Observations:

- slope f is neg, f' below x-axis
- slope f is pos, f' above x-axis
- inflection pts of f are turning pts for f'