§2.3: (Cont.)

Warm up: Compute the composition of the following: $f(g(x))$ and $g(f(x))$ where $f(x) = x^2 + x + 1$, $g(x) = \ln(x^2+1)$.

Sol:

$\ulcorner\ \ln(x^a) = a\ln(x)$

$(x+y)^a \neq x^a + y^a$

$\ln(x)^a \neq a\ln(x)\ \lrcorner$

$$f(g(x)) = f(\ln(x^2+1))$$

$$= (\ln(x^2+1))^2 + \ln(x^2+1) + 1$$

$\ulcorner\ (a+b)^2 = a^2 + 2ab + b^2\ \lrcorner$

$$g(f(x)) = g(x^2+x+1)$$

$$= \ln\left(\boxed{(\underbrace{x^2}_{a} + \underbrace{x+1}_{b})^2} + 1\right)$$

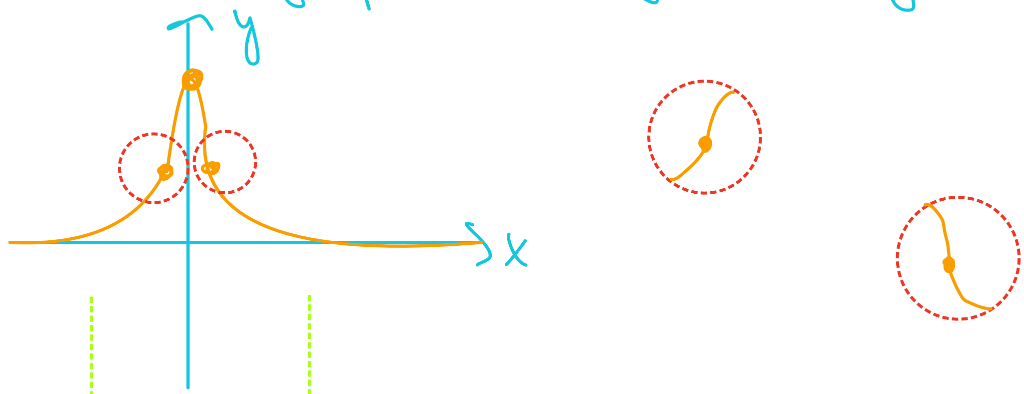
$$= \ln\left(\boxed{x^4 + 2x^2(x+1) + (x+1)^2} + 1\right)$$

$$= \ln\left(x^4 + \underline{2x^3} + \underline{2x^2} + \underbrace{x^2 + 2x + 1} + 1\right)$$

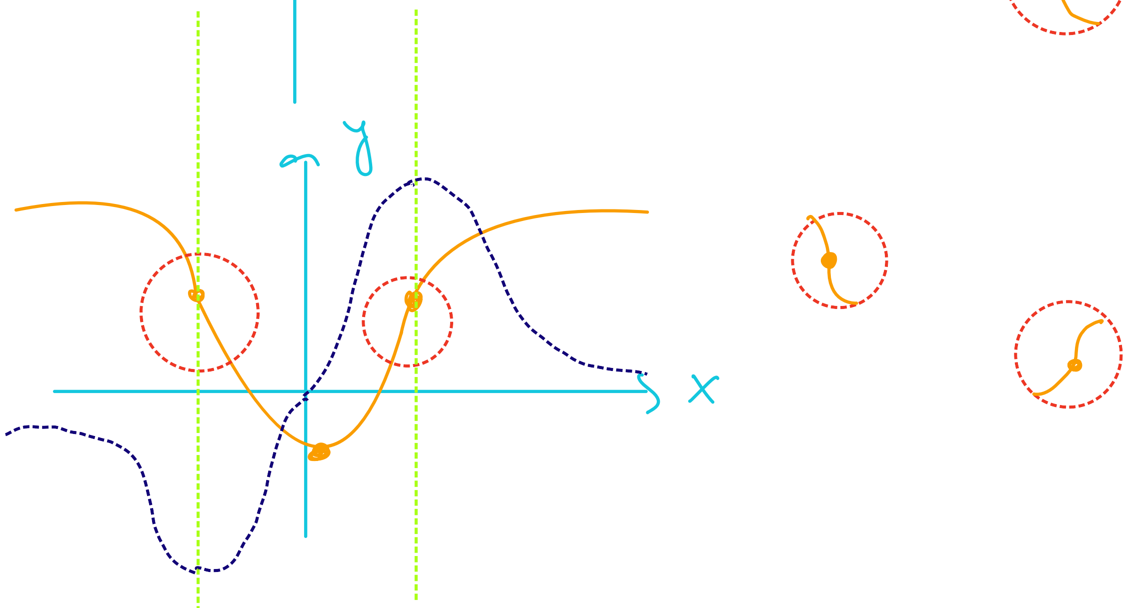
$$= \ln(x^4 + 2x^3 + 3x^2 + 2x + 2)$$

Ex: Sketch a graph of the derivative of the function whose graph is given by:

a.

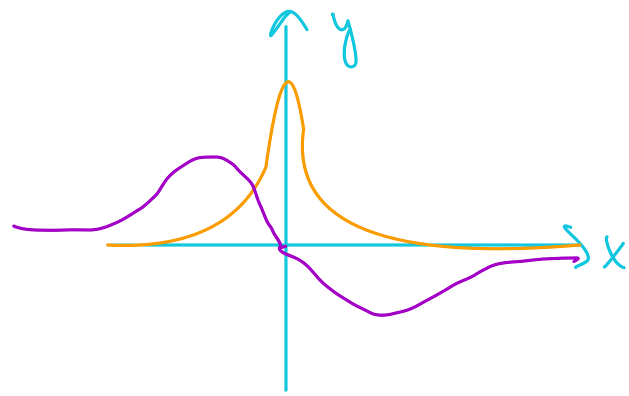


b.



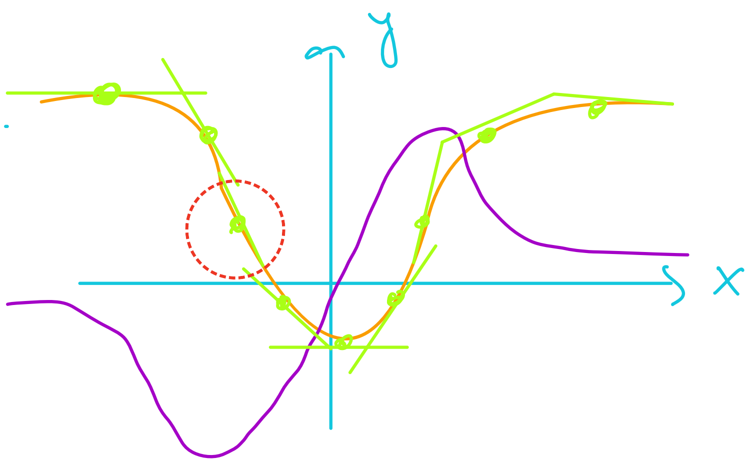
Sol:

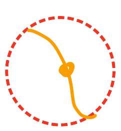
a.



* tangent line approx.*

b.



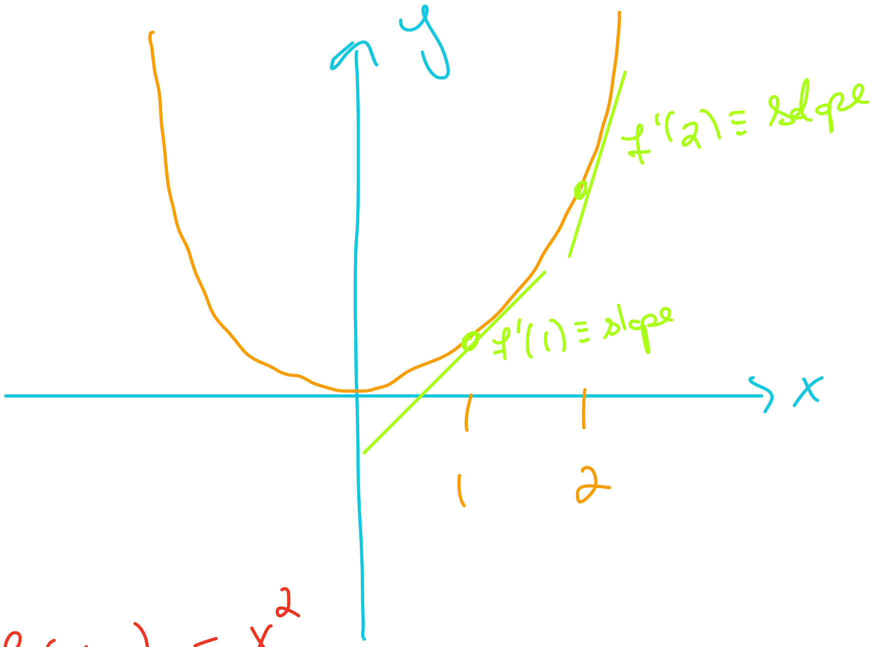


* local min/max
⇒ slope of tangent line is zero

* inflection pt
⇒ slope of tangent line switches direction

Ex: Approximate $f'(2)$ and $f'(1)$ where $f(x)=x^2$.

Sol:



$f(x) = x^2$

$f'(x) = 2x$

$f''(x) = 2$

$f'''(x) = 0$

§2.4: Second derivative

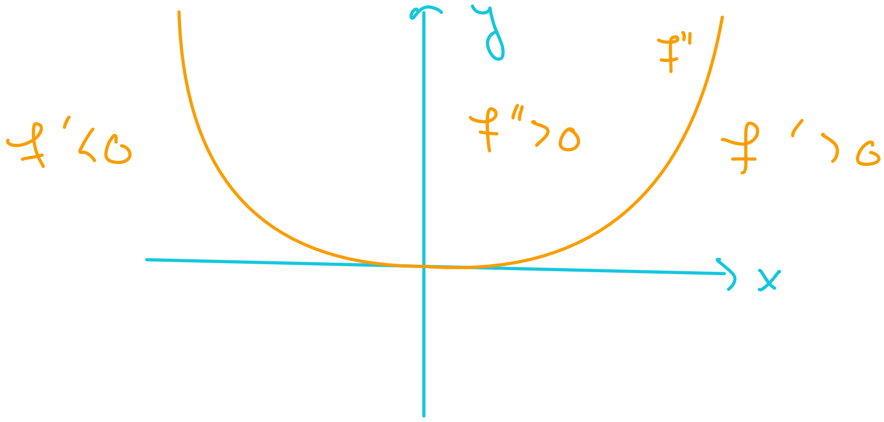
<u>Obj</u>: "Taking" the derivative of a function gives us a new function. Hence, we can "take" the derivative of a derivative.

<u>Def</u>: The "second derivative" of a function $y = f(x)$ is the derivative of the derivative of $f(x)$. We denote this by $\left[f'(x) = \frac{df}{dx} = \frac{d}{dx} f(x) \right]$

$$f''(x) \quad \text{or} \quad \frac{d}{dx}\left(\frac{d}{dx} f(x)\right) \quad \text{or} \quad \frac{d^2 f}{d^2 x}.$$

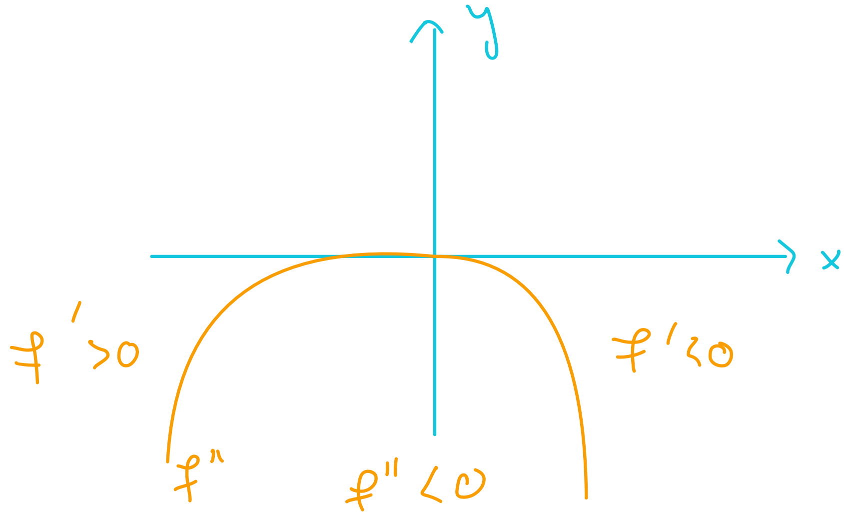
<u>Rmk</u>: Let I be an interval and f a function.

- $f' > 0$ on $I \Rightarrow f$ is increasing on I
- $f' < 0$ on $I \Rightarrow f$ is decreasing on I
- $f'' > 0$ on $I \Rightarrow f'$ is increasing on I
- $f'' < 0$ on $I \Rightarrow f'$ is decreasing on I.
- $f'' > 0$ on $I \Rightarrow f$ is concave up on I
- $f'' < 0$ on $I \Rightarrow f$ is concave down on I



* concave up *

[concavity means "where it opens"]



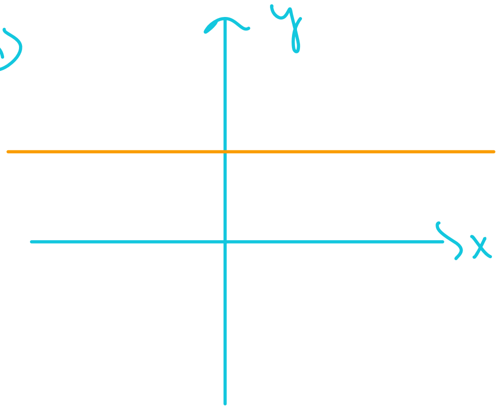
* concave down *

Problems for Chp 2:

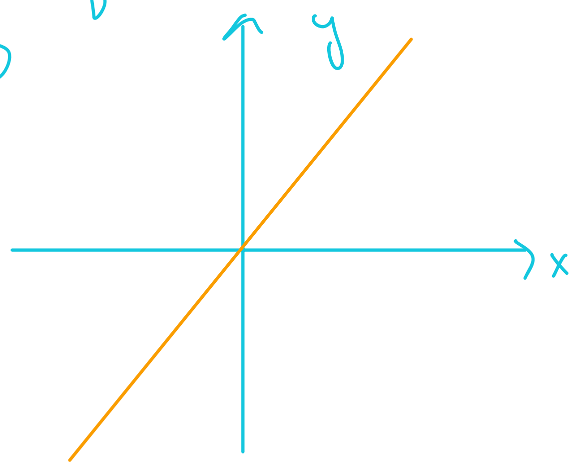
Rmk: The objective of Chp 2 is to introduce derivatives, and try to understand what they tell us geometrically about the function.

§2.2: Sketch a graph of $f'(x)$.

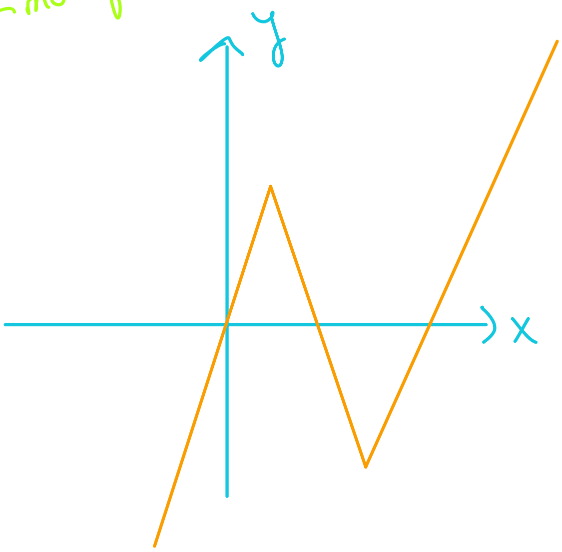
(12)



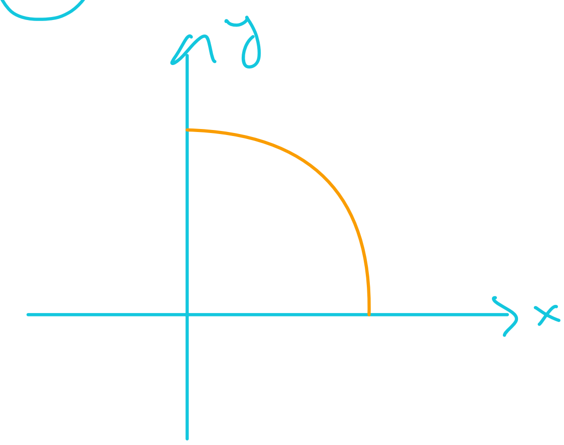
(13)



(14) ← modified



(16)



(17)