## §2: Focus on theory

35: If $f(x) = 3x - 2$, then show $f'(x) = 3$.

Sol:

More generally, if $f(x) = mx + b$, then we show $f'(x) = m$. Then

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \qquad (\text{def. of } f')$$

$$= \lim_{h \to 0} \frac{m(x+h) + b - (mx+b)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{mx} + mh + \cancel{b} - \cancel{mx} - \cancel{b}}{h}$$

$$= \lim_{h \to 0} \frac{m\cancel{h}}{\cancel{h}} = \lim_{h \to 0} m = m.$$

39: Show the following: If $f(x) = 5x^2 + 1$, then $f'(x) = 10x$.

Sol:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \qquad (\text{def})$$

$$= \lim_{h \to 0} \frac{5(x+h)^2+1-(5x^2+1)}{h}$$ (plug in x+h, x to f)

$$= \lim_{h \to 0} \frac{5(x^2+2xh+h^2)+1-5x^2-1}{h}$$

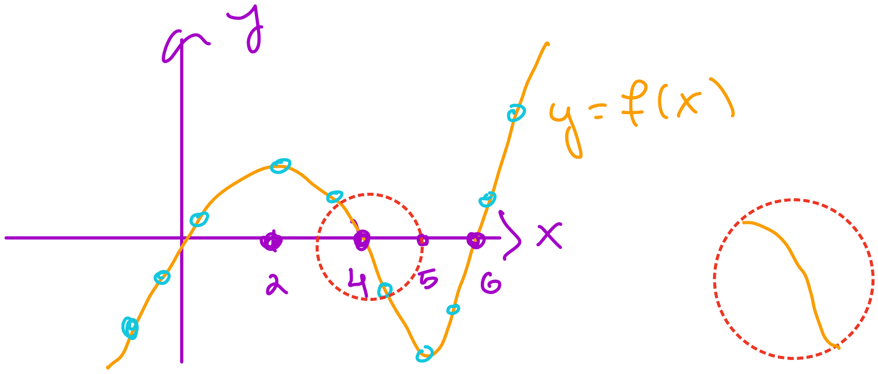
$$= \lim_{h \to 0} \frac{\cancel{5x^2}+10xh+5h^2 \cancel{+1} \cancel{-5x^2} \cancel{-1}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(10x+5h)}{\cancel{h}}$$ (reduce)

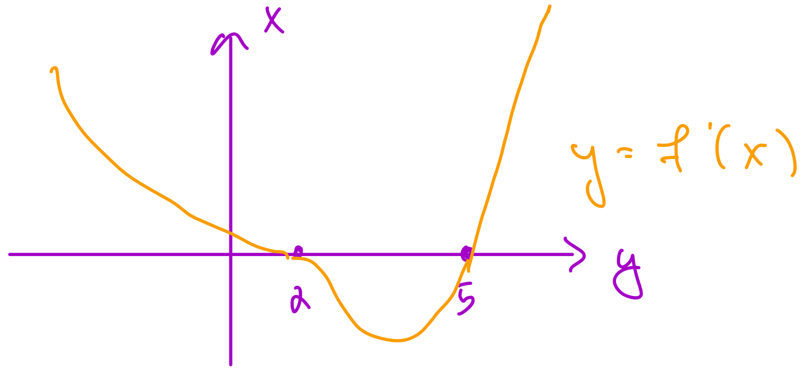
$$= \lim_{h \to 0} 10x+5h = 10x$$

§2.2:

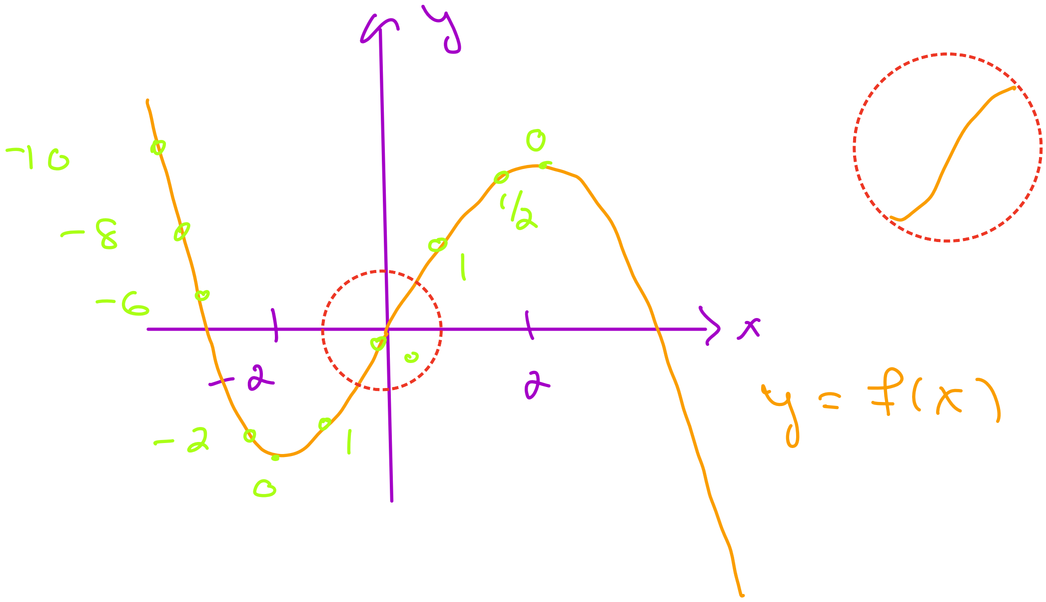
14: Sketch a graph of $f'(x)$ where the graph of $f$ is given by



Sol :

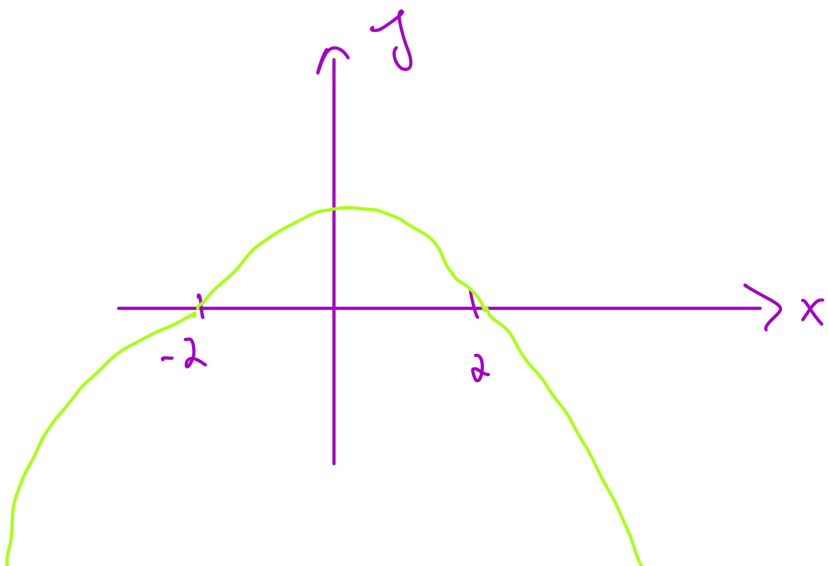



18: Sketch a graph of $f'(x)$ where the graph of $f$ is given by




* values are not precise!

Sol :



15: * Look at this on your own *

Note: 37, 38 will not be on exam 1

§1.6:

18: Solve for t where $2e^t - 5 = 0$.

Sol: * See summary at end §1.6 *

$$2e^t - 5 = 0 \implies 2e^t = 5$$

$$\implies e^t = \frac{5}{2}$$

$$\implies \ln(e^t) = \ln\left(\frac{5}{2}\right)$$

$$\implies t\ln(e) = \ln\left(\frac{5}{2}\right)$$

$$\implies t = \ln(5/2) \quad (\ln(e) = 1)$$

[ $\log_a(a^x) = x$

$\ln(x) := \log_e(x)$ ]

17: Solve for t where $Ae^{2t} = Be^t$ for $A, B \in (0, \infty)$.

Sol: $Ae^{2t} = Be^t \implies e^{2t} = \frac{B}{A}e^t$

$\left[ \frac{x^a}{x^b} = x^{a-b} \right]$

$$\Rightarrow \boxed{\frac{e^{2t}}{e^t}} = \frac{B}{A}$$

$$\Rightarrow e^{2t-t} = \frac{B}{A}$$

$$\Rightarrow e^t = \frac{B}{A}$$

$$\Rightarrow t = \ln(e^t) = \ln(B/A)$$

Alt: $Ae^{2t} = Be^t \Rightarrow \ln(Ae^{2t}) = \ln(Be^t)$

* both sides are products,

$$\Rightarrow \ln(A) + \ln(e^{2t}) = \ln(B) + \ln(e^t)$$

$\ln(ab) = \ln(a) + \ln(b)$

$$\Rightarrow \ln(A) + 2t\,\overset{1}{\cancel{\ln(e)}} = \ln(B) + t\,\overset{1}{\cancel{\ln(e)}}$$

$$\Rightarrow \ln(A) + 2t = \ln(B) + t$$

* Note $\ln(a+b) \neq \ln(a) + \ln(b)$

$$\Rightarrow 2t - t = \ln(B) - \ln(A)$$

$$\Rightarrow t = \ln\left(\frac{B}{A}\right).$$

$\ln(x)$ $\quad e^x$ $\quad e^1 = e$

$\ln(e^x) = x, \quad e^{\ln(x)} = x$