① Show: $f(x) = x - x^2 \Longrightarrow f'(x) = 1 - 2x$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

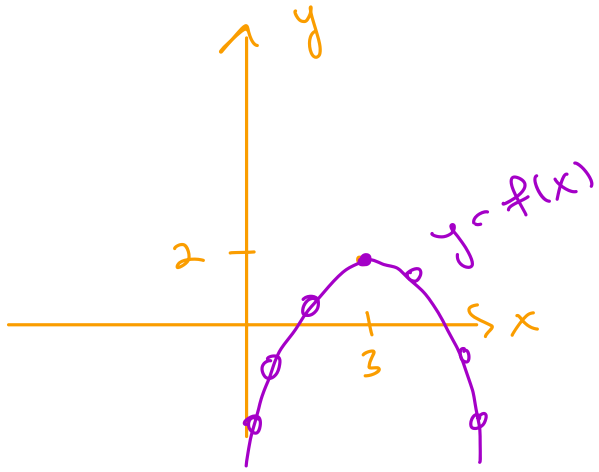
$$= \lim_{h \to 0} \frac{\overbrace{\left((x+h) - (x+h)^2\right)}^{f(x+h)} - \overbrace{(x - x^2)}^{f(x)}}{h}$$

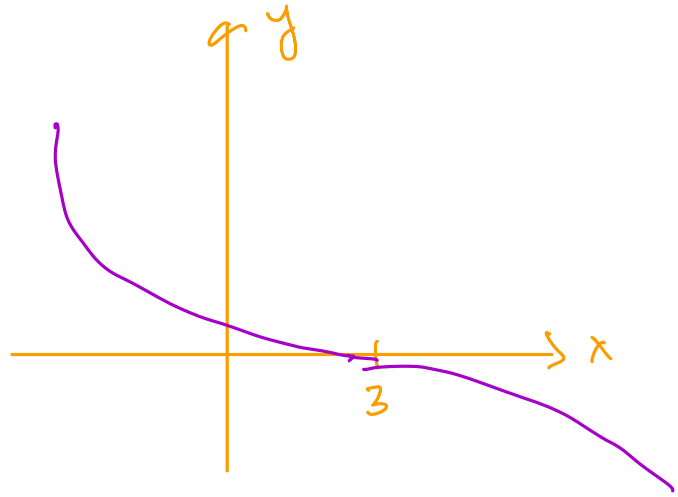
$$= \lim_{h \to 0} \frac{\cancel{x} + h - \cancel{x^2} - 2xh - h^2 - \cancel{x} + \cancel{x^2}}{h}$$

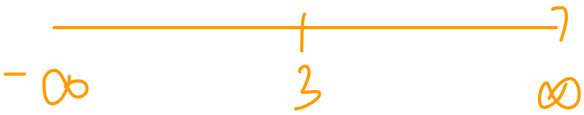
$$= \lim_{h \to 0} \frac{h - 2xh - h^2}{h} = \lim_{h \to 0} \frac{\cancel{h}(1 - 2x - h)}{\cancel{h}}$$

$$= \lim_{h \to 0} 1 - 2x - h = 1 - 2x.$$

② Graph the der. of $f(x) = -(x-3)^2 + 2$







(3) $f(x) = \sqrt{x+4}$, $g(x) = x^2$

$f \circ g$, $g \circ f$, $f \circ f$

$$f(g(x)) = f(x^2) = \sqrt{x^2+4}$$

$$g(f(x)) = g(\sqrt{x+4}) = (\sqrt{x+4})^2 = x+4$$

$$f(f(x)) = f(\sqrt{x+4}) = \sqrt{\sqrt{x+4}+4}$$

(4) Solve for t where $Pe^{4t} - Qe^{-t} = 0$.

$$Pe^{4t} - Qe^{-t} = 0 \implies Pe^{4t} = Qe^{-t}$$

$$\implies e^{4t} = \frac{Q}{P} e^{-t}$$

$$\implies e^{4t} e^{t} = \frac{Q}{P} \cancel{e^{-t}} \cdot \cancel{e^{t}}$$

$$\implies e^{5t} = \frac{Q}{P}$$

$$\implies \ln(e^{5t}) = \ln\left(\frac{Q}{P}\right)$$

$$\implies 5t = \ln\left(\frac{Q}{P}\right)$$

$$\Longrightarrow t = \frac{\ln\left(\frac{Q}{P}\right)}{5}. \square$$

⑤ $\frac{\Delta y}{\Delta x}$ for $f(x) = 2x^2$ on $[1,3]$

$$\frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b-a} = \frac{f(3) - f(1)}{3-1}$$

$$= \frac{2\cdot 3^2 - 2\cdot 1^2}{2} = \frac{18-2}{2} = \frac{16}{2} = 8. \square$$

⑥ line from $(-2, 1)$ and $(2, 3)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-1}{2-(-2)} = \frac{2}{4} = \frac{1}{2}$$

$$y - y_i = m(x - x_i) \quad (i = 1 \text{ or } 2)$$

$$y - 3 = \frac{1}{2}(x-2) \Rightarrow \quad (\text{case } (x_2, y_2) = (2,3))$$

$$y = \frac{1}{2}x - \frac{2}{2} + 3 = \frac{1}{2}x + 2$$

$$\therefore y = \frac{1}{2}x + 2. \square$$

(7) slope, y-int from $-4y+2x+8=0$

$$-4y+2x+8=0 \Rightarrow -4y=-2x-8$$

$$\Rightarrow y=-\frac{1}{4}(-2x-8)$$

$$=\frac{1}{4}(2x+8)$$

$$=\frac{1}{2}x+\frac{8}{4}=\frac{1}{2}x+2$$

$(=mx+b)$

$$\Rightarrow m=\frac{1}{2}, \text{ y-int at } 2. \blacksquare$$

(8) $f(x)=ax^2+bx+c \Rightarrow f'(x)=2ax+b$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\left(a(x+h)^2+b(x+h)+c\right)-(ax^2+bx+c)}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{ax^2}+a2xh+ah^2+\cancel{bx}+bh+\cancel{c}-\cancel{ax^2}-\cancel{bx}-\cancel{c}}{h}$$

$$=\lim_{h\to 0}\frac{2axh+ah^2+bh}{h}=\lim_{h\to 0}\frac{\cancel{h}(2ax+ah+b)}{\cancel{h}}$$

$$= \lim_{h \to 0} 2ax + ah + b$$

$$= 2ax + b \,. \quad \blacksquare$$