Math 122
Spring 2024
Exam 3 Practice
4/11/2024
Time Limit: 75 Minutes

Name: SOLUTIONS

Signature: ______________________

This exam has 8 questions, for a total of 80 points and 0 bonus points. Unless otherwise specified, there is no form of technology allowed. Further, final solutions must be written in the prescribed boxes, and all work must be shown. There is paper provided in the front of the class for scratch work. Any numerical values given for a final answer must be precise. *The actual format of the exam is not a direct reflection of this practice.*

Grade Table (for teacher use only)

Question	Points	Bonus Points	Score
1	10	0	
2	10	0	
3	10	0	
4	10	0	
5	10	0	
6	10	0	
7	10	0	
8	10	0	
Total:	80	0	

1. (10 points) If a is a constant, find all critical points of $f(x) = 5ax - 2x^2$. Find the value of a so that f has a local maximum at $x = 6$.

Critical points:

$f'(x) = 5a - 4x$

$5a - 4x = 0 \Rightarrow 5a = 4x \Rightarrow \boxed{\frac{5}{4}a = x}$ this is the only critical point.

If we want the critical point at $x = 6$, then

$\frac{5}{4}a = 6 \Rightarrow 5a = 24 \Rightarrow \boxed{a = \frac{24}{5}}$

To show this is a local maximum, notice

$f''(x) = -4$, so $f''(6) = -4$, and therefore, there is a local maximum at $x = 6$.

2. (10 points) Find all critical points of the function $g(x) = x^4 - 2x^2$. Determine which of these are local minimums, local maximums, inflection points, or neither.

$g'(x) = 4x^3 - 4x$

$4x^3 - 4x = 0 \Rightarrow 4x(x^2 - 1) = 0 \Rightarrow 4x(x+1)(x-1) = 0$

So the critical points are $x = 0, x = 1, x = -1$.

$g''(x) = 12x^2 - 4$

$g''(0) = -4$, so there is a local max at $x = 0$

$g''(1) = 8$, so there is a local min at $x = 1$

$g''(-1) = 8$, so there is a local min at $x = -1$

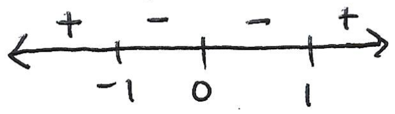
3. (10 points) Find all global minimum and maximums of $f(x) = x + \frac{1}{x}$ for $x > 0$.

$f'(x) = 1 - 1/x^2$

$1 - 1/x^2 = 0 \Rightarrow 1 = 1/x^2 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1.$

Also, $f(x)$ is not defined at $x=0$, so $x=0$ is a CP.

Critical points: $x=-1$, $x=0$, $x=1$



So $x=1$ is a local min.

In fact, $x=1$ is the global minimum on $x>0$.

There is no global maximum on the interval $x>0$.

$f'(-2) = 1 - 1/4 = 3/4 > 0$

$f'(-0.5) = 1 - 4 = -3 < 0$

$f'(0.5) = 1 - 4 = -3 < 0$

$f'(2) = 1 - 1/4 = 3/4 > 0$

4. (10 points) Compute $\int (5x - \sqrt{x})dx$.

$$\int (5x - \sqrt{x})dx$$

$$= \int 5x\,dx - \int x^{1/2}dx$$

$$= \boxed{\frac{5}{2}x^2 - \frac{2}{3}x^{3/2} + C}$$

5. (10 points) Compute $\int \frac{1}{z^3} dz$.

$$\int \frac{1}{z^3} dz = \int z^{-3} dz$$
$$= \boxed{-\tfrac{1}{2} z^{-2} + C}$$

6. (10 points) Compute $\int (\frac{3}{t} - \frac{3}{t^2}) dt$.

$$\int \left(\tfrac{3}{t} - \tfrac{3}{t^2}\right) dt$$
$$= \int \tfrac{3}{t}\, dt - \int 3t^{-2} dt$$
$$= 3\int \tfrac{1}{t}\, dt - 3\int t^{-2} dt$$
$$= 3\ln|t| - 3(-t^{-1}) + C$$
$$= \boxed{3\ln|t| + 3/t + C}$$

7. (10 points) Compute $\int_1^2 \frac{1}{x^2} dx$.

$\int_1^2 1/x^2 \, dx$

$= \int_1^2 x^{-2} dx$

$= -x^{-1} \Big]_1^2$

$= -1/x \Big]_1^2$

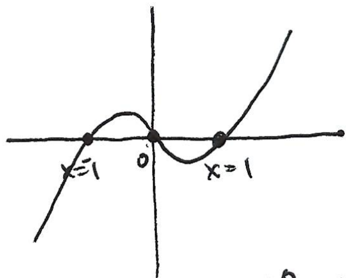
$= -1/2 - (-1/1) = \boxed{1/2}$

8. (10 points) Find the area between the x-axis and the graph of $x^3 - x$.

$f(x) = x^3 - x$

Get an idea of what f(x) looks like

$f(x) = 0 \Leftrightarrow x^3 - x = 0 \Leftrightarrow x(x^2 - 1) = 0 \Leftrightarrow x(x+1)(x-1) = 0$

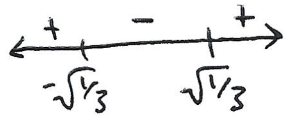


$f'(x) = 3x^2 - 1 = 0$

$3x^2 = 1$

$x^2 = 1/3$

$x = \pm\sqrt{1/3}$



$f'(-1) = 2$

$f'(0) = -1$

$f'(1) = 2$

area bounded is $\int_{-1}^0 (x^3 - x)\,dx + -\int_0^1 (x^3 - x)\,dx$

$= \left[x^4/4 - x^2/2\right]_{-1}^0 - \left[x^4/4 - x^2/2\right]_0^1 = -(1/4 - 1/2) - (1/4 - 1/2) = \boxed{1/2}$