

$$1. f(x) = x^3 - ax$$

$$f'(x) = 3x^2 - a \quad (\text{poly, so dom } f' = \mathbb{R})$$

$$f'(x) = 0 \Rightarrow 3x^2 - a = 0$$

$$\Rightarrow 3x^2 = a$$

$$\Rightarrow x^2 = \frac{a}{3}$$

$$\Rightarrow \boxed{x = \pm \sqrt{\frac{a}{3}}}$$

$$x = \pm a \Rightarrow \pm a = \pm \sqrt{\frac{a}{3}}$$

$$\Rightarrow \boxed{a = 12}$$

$$2. g(x) = x^5 - 5x^4 + 35$$

$$g'(x) = 5x^4 - 20x^3 \\ = 5x^3 \cdot (x - 4)$$

$$g'(x) = 0 \Rightarrow x = 0, 4$$

$$g''(x) = 20x^3 - 60x^2$$

$$g''(0) = 0 \Rightarrow \text{2}^{\text{nd}} \text{ der. test in incl.}$$

$$g''(4) = 20 \cdot 4^3 - 60 \cdot 4^2 \\ = 4^2 (20 \cdot 4 - 60)$$

$$= 4^2 \cdot 20 \cdot (4-3) > 0$$

\therefore 2nd der. test \Rightarrow local min at 4

$$g''(x) = 0 \Rightarrow 20x^3 - 60x^2 = 0$$

$$\Rightarrow 20x^2(x-3) = 0$$

$$\Rightarrow x = 0, 3$$

$-1 < 0 < 1$:

$$g''(-1) = 20(-1)^2(-1-3) < 0$$

$$g''(1) = 20 \cdot 1^2 \cdot (1-3) < 0$$

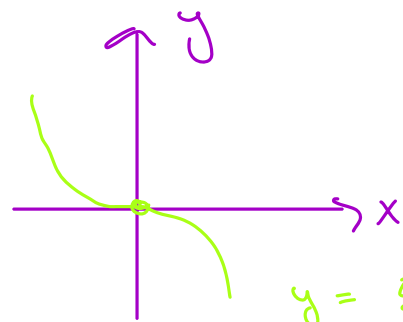
$\therefore 0$ not inf. pt.

$2 < 3 < 4$: $g''(2) = 20 \cdot 2^2 \cdot (2-3) < 0$

$$g''(4) = 20 \cdot 4^2 \cdot (4-3) > 0$$

$\therefore 3$ is inf. pt.

We use 1st der. test to see if 0 is a local extrema:



$$y = 5x^4 - 20x^3 = f'(x)$$

f' changes pos. to neg. at 0 \Rightarrow

by 1st der. test, $\boxed{0}$ in local max.

3. $f(t) = \frac{t}{t^2+1}$

$$f'(t) = \frac{1 \cdot (t^2+1) - t(2t)}{(t^2+1)^2}$$

$$= \frac{t^2+1-2t^2}{(t^2+1)^2} = \frac{1-t^2}{(t^2+1)^2}$$

$$f'(t) = 0 \iff 1-t^2 = 0 \iff t = \pm 1$$

$$f''(t) = \frac{2t(t^2-3)}{(t^2+1)^3}$$

$$f''(1) = \frac{2 \cdot 1 \cdot (1-3)}{2^3} < 0 \Rightarrow \boxed{1 \text{ local max}}$$

$$f''(-1) = \frac{2 \cdot (-1) \cdot (-1-3)}{2^3} > 0 \Rightarrow \boxed{-1 \text{ local min}}$$

$$4. \int (t^3 - \frac{t^2}{2} - t) dt =$$

$$\int t^3 dt - \frac{1}{2} \int t^2 dt - \int t dt =$$

$$\frac{1}{4} t^4 - \frac{1}{2} \cdot \frac{1}{3} t^3 - \frac{1}{2} t^2 + C =$$

$$\boxed{\frac{1}{4} t^4 - \frac{1}{6} t^3 - \frac{1}{2} t^2 + C}$$

$$5. \int (\sqrt{z})^3 dz = \int z^{\frac{3}{2}} dz$$

$$= \frac{1}{\left(\frac{3}{2}\right)} z^{\frac{3}{2}+1} + C$$

$$\boxed{= \frac{2}{3} z^{\frac{5}{2}} + C}$$

$$6. \int \left(\frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} \right) dx =$$

$$\int \frac{1}{x} dx + \int \frac{1}{x^2} dx + \int \frac{1}{x^3} dx =$$

$$\boxed{\ln|x| - x^{-1} - \frac{1}{2} x^{-2} + C}$$

$$7. \int_1^4 \frac{1}{\sqrt{x}} dx = \int_1^4 x^{-\frac{1}{2}} dx$$

$$= 2\sqrt{x} \Big|_{x=1}^{x=4} = 2\sqrt{4} - 2\sqrt{1}$$

$$= 4 - 2 = \boxed{2}$$